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# ON THE CONAF EVALUATION MODEL

Alan F. Karr

August 1976



INSTITUTE FOR DEFENSE ANALYSES  
PROGRAM ANALYSIS DIVISION

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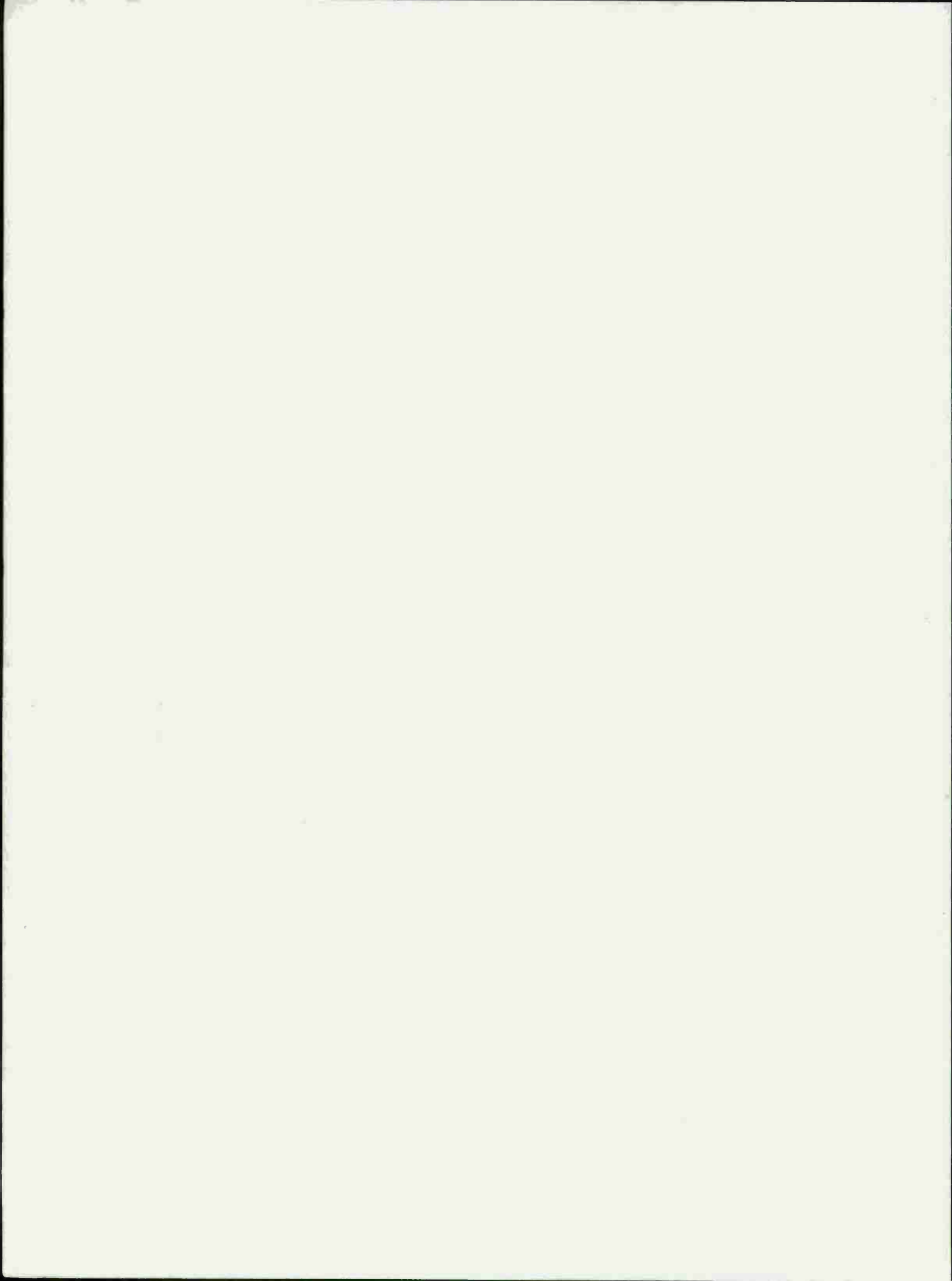
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## 1. INTRODUCTION

This paper is a review and critique of the CONAF Evaluation Model (CEM), which has been developed by the General Research Corporation. The major goal of the paper is identification of major assumptions underlying the model and of the implications--both mathematical and physical--of those assumptions. In particular, we give (in Section 5) a detailed treatment of the mathematical assumptions underlying the attrition equations used in the CEM. The reader is cautioned that there are other important criteria for model evaluation (e.g., consistency with historical data, intuitive plausibility, computational feasibility and simplicity) that are given less weight in our evaluations and criticisms. That such criteria are important cannot be doubted; the competence and tastes of the author, however, are in the logical and mathematical aspects of modeling. Moreover, use of assumptions as a basis for comparing and evaluating models is a well-recognized and important technique. Similar analyses of the Vector-I and Lulejian-I combat models are presented in References [7] and [8], respectively; the IDAGAM I combat model is described in References [1] and [3]. The main source of information used in preparing this paper is Reference [9].

The CEM is a computerized, deterministic simulation of bilateral, non-nuclear, theater-level combat in which a continuous FEBA exists. Its main purposes are comparison of different force structures by means of FEBA displacement, resource expenditure, time consumption, and force evolution during the campaign; and analysis of the effect of command

decisions on missions of subunits of each force and on allocation of supporting resources such as artillery and CAS aircraft. In particular (Reference [9, p. 90]) the model is suggested for studying the effect on combat results of resource resupply policies and rates and variations of the mix between combat units and resources providing support fire.

The main feature that distinguishes the CEM from comparable models such as Vector-I, Lulejian-I, and IDAGAM I (References [7], [8], and [1] and [3], respectively) is a complicated representation of hierarchical combat decision making, in which decisions on each organization level (e.g., division) are made in light of previously determined decisions, principally overall mission selection, reserve commitment, and support fire allocation by higher level organizations (e.g., armies and corps). Moreover, lower level decisions may be changed more frequently than those taken at higher levels. A more complete description of the decision-making process appears in Section 7 of this paper. Secondary emphases of the model are on renewable resources (e.g., the representation of damaged but repairable tanks) and on detailed methods of resource accounting (see Section 3). Less effort than is desirable seems to have been devoted to attrition equations, to computation of FEBA movement and, especially, to modeling of the entire air combat process.

## 2. GEOGRAPHY

The method of representation of battlefield geography used in the CEM is standard. There is a piecewise linear FEBA separating the two sides and no encirclements are permitted (i.e., the piecewise linear FEBA is interpreted as an approximation to a continuous FEBA). The battlefield is divided, orthogonal to the FEBA, into fundamental units called *minisectors*, of which at most 1,000 are permitted. There is a division, parallel to the FEBA, of each minisector into terrain intervals of at least 0.1 kilometer in depth. In terms of actual geography, minisector widths may vary; see Figure 1.

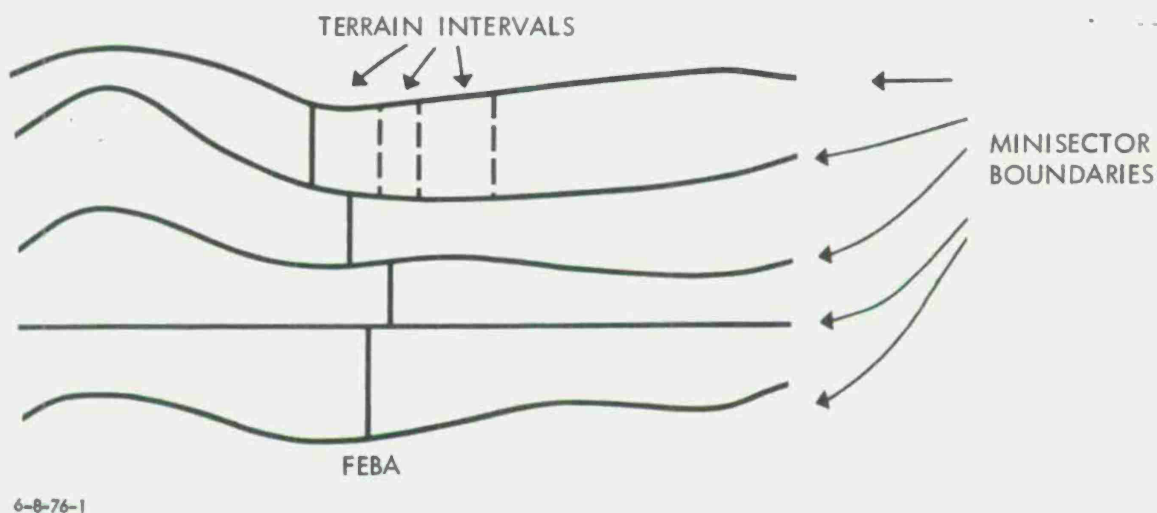


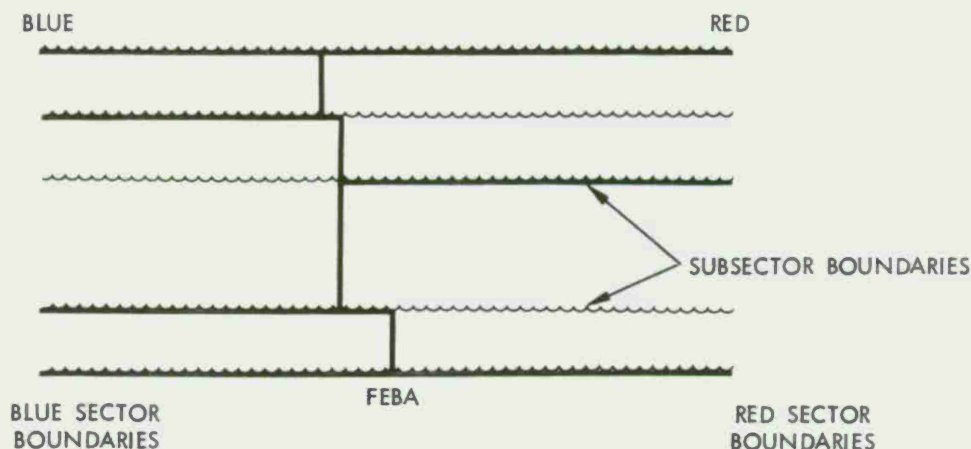
Figure 1. BATTLEFIELD REPRESENTATION IN CEM

There are four types of terrain, namely:

- (1) flat land on which tanks can operate without restriction,
- (2) more rugged land on which tanks can operate, but only marginally,
- (3) land on which tanks are confined to roads,
- (4) major obstacles.

Treatment of major obstacles is described in slightly more detail in Section 6.

As discussed in Section 3, the level of force resolution is that of division on the Red side and brigade on the Blue side. A Red *sector* is, therefore, the front occupied by one Red division, while a Blue *sector* is the front occupied by one Blue brigade. Sector boundaries must coincide with minisector boundaries, but need not be the same on both sides of the FEBA; sector boundaries on each side may change during the course of the campaign (see Section 7). *Subsectors* are artificial divisions, running the full depth of the battlefield, obtained by extending all sector boundaries, as in Figure 2.



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Figure 2. SECTORS AND SUBSECTORS IN CEM

In Figure 2 we have shown FEBA position as constant on each subsector. Section 6 confirms this constancy. While artificial in the geographic and organizational senses, subsectors are the most important battlefield divisions in the CEM. No combat is assumed to occur across subsector boundaries and all attrition computations and FEBA movement calculations are made on the basis of subsectors.

Across a subsector front, therefore, part (possibly all) of a Blue brigade and part of a Red division face each other. Forces are always distributed uniformly over the minisectors comprising a subsector; indeed, unless over the entire front the force density per minisector is essentially constant, difficulties arise with the principal personnel attrition computation in the model (see Section 5).

FEBA movement is in multiples of 0.1 kilometer, as described in more detail in Section 6. If the FEBA is moving sufficiently slowly, prepared defenses may be erected (see Reference [9, pp. 36-37] for details). The use of exponential smoothing (with respect to time) in this context seems questionable, but is probably of minor consequence. Rear regions are included in the model, but not very explicitly.



### 3. RESOURCES AND RESOURCE ACCOUNTING

In terms of resources and resource accounting the CEM differs from comparable models (Vector-I, Lulejian-I, and IDAGAM I) in several fundamental respects. The most obvious is a seeming asymmetry of force resolution. Blue forces are resolved to the brigade level, but Red forces are resolved only to the division level. One is apt, on purely aesthetic and theoretical grounds, to condemn such asymmetry as introducing unnecessary errors that cannot be estimated *a priori*, even qualitatively. That is, on an abstract level, the asymmetry of the model is an underlying assumption that may have an overwhelming effect on the results of the model. Furthermore, the nature and magnitude of the deviation of such results from those obtained from a symmetric model are most difficult to predict, especially in a quantitative sense.

The situation is rather different, however, when viewed in terms of the principal intended use of the model: evaluation of NATO force structures and decision-making policies against a potential Warsaw Pact enemy in Europe. For this case, a Blue brigade and a Red division are more nearly equal numerically than are a Blue division and a Red division, so that the asymmetry of the model merely reflects an asymmetry in force structure that actually exists in the potential situation the model is intended to analyze. Moreover, the model is in this context numerically symmetric in that resolution is to the level of force units of approximately equal size on both sides. Even so, there remains an asymmetry in the decision-making process in the model, with the Blue side having four

levels of decision making but the Red side only three. For example (see Section 7 for details), Blue brigade boundaries can be changed more often than Red division boundaries. This flexibility means that Blue is better able to adjust to the evolving status of the combat. On balance, we feel that with regard to asymmetry in decision-making processes, the resultant errors are not substantially larger than those resulting from other main underlying assumptions of the model; we again emphasize, though, that even qualitative prediction of these errors is difficult.

Finally (with reference to asymmetry), potential users of the model should be cautioned that if it were used to analyze situations in which Blue and Red units of the same name *are* numerically equal, substantial--and, again, unpredictable--errors may result. The CEM cannot, despite any other virtues it possesses, be recommended for such analyses.

A second fundamental difference between the CEM and comparable models is the accounting for resources in terms of unit status files rather than in terms of numbers of resources in various categories. For each Blue brigade and cavalry unit, each Red division, and each divisional artillery battalion on either side, there is maintained in the model a *status file* which is periodically updated to reflect losses, reinforcements, position, supply consumption, and so on. These status files are essentially the only means of resource accounting in the CEM. Nondivisional artillery on each side is grouped into a single status file; aircraft are accounted for by type. Advantages of the CEM scheme of resource accounting are that it allows large numbers of battalion structures and weapon types and it incorporates the detail necessary for the decision-making representation in the model. Disadvantages are the input and storage requirements. There is no clear-cut preference for or against the CEM methodology; it is worthwhile that

models exist with different resource accounting methods that may be suited to different specific applications.

Specifically, the resources represented in the CEM are--

- approximately 50 types of Blue maneuver battalion (or Red regiments),
- eight types of artillery, which may be in up to 15 types of divisional artillery battalions,
- twelve types of tanks,
- twelve types of armored personnel carriers,
- helicopters of up to five types, in division and corps cavalry units,
- twelve types of mortars and antitank weapons,
- two types of tactical aircraft.

The number of different battalion *types* represented in a given run of the model restricts the actual number of battalions that can be modeled.

Two types of aircraft are represented in the CEM: air defense fighters (ADF), which function in the intercept mission, and tactical fighters (TF), which are allocated among counter-air, close air support and reconnaissance/interdiction missions in a manner described in Section 8. No special-purpose aircraft that are adapted to particular missions are included in the model; this lack is, we feel, a significant shortcoming of the CEM.

Each side is organized in the following manner. The theater forces are divided into field armies, each of which consists of 1 to 5 corps. Each corps contains 1 to 5 divisions, corps artillery battalions, and a corps cavalry unit. Each Blue division contains two or three brigades, together with divisional artillery battalions and a divisional cavalry unit; Red divisions are not further divided (except into maneuver battalions for accounting purposes), but each has associated divisional artillery battalions and a cavalry unit. Every

Blue brigade is composed of a number of maneuver battalions. A given unit of division size or larger may have in reserve at most one unit of the next smaller size. Reinforcements that would violate constraints on the organization sizes noted above lead to creation of new units. For example, if commitment of a reinforcing division would create a corps of six divisions, two new three-division corps are created and the original corps' frontage is split between them.

Artillery battalions associated with Blue brigades and Red divisions function in the direct support (DS) role. Divisions, corps and armies may have, in addition, general support (GS) artillery battalions that can function in the DS role, in counterbattery fire against enemy DS artillery battalions, or in fire against enemy maneuver units in reserve. Firepower from DS artillery battalions appears in computation of losses and FEBA movement; see Sections 4 and 5 for details.

The CEM represents four logistic functions: supply, maintenance, personnel care, and transportation, but only in highly aggregated form. The five categories of supplies are POL, ammunition, major weapons, personnel, and "other supplies." Maintenance support is represented by fractions of repairable vehicles, maintenance facility capacity and times required for repair. Capacities and rates of delivery for each class of supplies are used to represent transportation. Some wounded personnel are taken to theater hospitals, treated and then returned to the battle. Hospital capacities are also represented. As noted in Section 5 we believe that explicit consideration of "renewable resources" is a useful contribution of the model. Details of resupply modeling are given in Section 7.

For each combat unit there is defined a time dependent scalar measure of unit status known as its *state*, which is

given by

$$(1) \quad S_n = 100(F_n/F_0) ,$$

where

$S_n$  = state at time  $n$ ,

$F_n$  = unit firepower at time  $n$ ,

$F_0$  = unit TOE firepower.

For details of the computation of  $F_0$  and  $F_n$  (the latter reflects supply shortages and exposed flanks in particular) we refer to the next section. The state  $S_n$  at time  $n$  is computed from the unit status file and is used in mission selection (see Section 7) and in one attrition computation (see Section 5); it does not include support fire (if any) allocated to the unit in question. Concerning Equation (1) as a measure of unit status, we note that it is linear; analogous measures in comparable models are not. However, the state is used essentially only in modeling of decision processes and not in the main attrition computations.

Time in the model is discrete. The basic unit of time is the division time period (generally regarded as half a day). Corps, army, and theater time periods are multiples of the division time period (for example, two, four, and eight division time periods, respectively). Over the time period corresponding to a given echelon, decisions at that echelon concerning mission selection, allocation of fire support, and commitment or reconstitution of reserves cannot be modified. Decisions at each echelon are constrained by the current combat situation and currently effective decisions made at higher echelons. For further details the reader is referred to Section 7.



#### 4. FIREPOWER COMPUTATION

For purposes of computation of attrition and FEBA movement, firepower potential is grouped into 18 classes according to source classes:

- (1) hard ground sources (tanks)
- (2) medium ground sources (light armored vehicles)
- (3) soft ground sources (unarmored weapons)
- (4) artillery
- (5) helicopters
- (6) CAS aircraft;

and according to target classes:

- antitank potential
- anti-light armor potential
- antipersonnel potential.

Let  $ATP(i,k)$  [ $ALP(i,k)$ ,  $APP(i,k)$ , respectively] denote the antitank potential [anti-light armor potential, antipersonnel potential, respectively] of side  $k$  arising from source  $i$ . These basic potentials are simply linear combinations of per-weapon potentials; that is, for example,

$$(2) \quad ATP(i,k) = \sum_{\substack{j: \text{ weapon type } j \\ \text{is in class } i}} ATP(i,j,k)N(j,k) ,$$

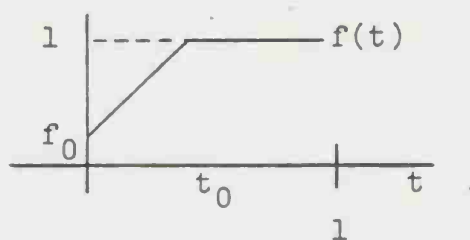
where

$ATP(i,j,k)$  = antitank potential of one weapon of type  $j$   
(in class  $i$ ) on side  $k$ ,

$N(j,k)$  = number of weapons of type  $j$  on side  $k$ .

This calculation is done for each subsector (see Section 2).  $ALP(i,k)$  and  $APP(i,k)$  are computed analogously.

Before attrition and FEBA movement calculations are performed, firepower potentials are modified to account for supply shortages and exposed flanks. A side with long exposed flanks is penalized by a diversion of firepower as well as by FEBA retraction procedures; however, no encirclements are permitted. To represent the effect of supply shortages, a normal supply consumption which depends on the type of engagement is computed (see Section 6); and potentials from ground sources are multiplied by a function  $f$  of the ratio  $A/N$  of actual consumption  $A$  to normal consumption  $N$ , which function is of the form



The cases  $t_0 = 1$ ,  $f_0 = 0$  are not excluded. Determination of actual consumption  $A$  of supplies is done using the equation

$$(3) \quad A = \begin{cases} N & \text{if } \frac{S}{N} \geq a \\ S & \text{if } \frac{S}{N} < a, \end{cases}$$

where

$S$  = supplies on hand,

$a$  = safety level (an input).

The units of  $a$  are (division) time periods.

Helicopters are constrained by supply shortages in that only as many fly as can be fully loaded with POL and ammunition. Artillery firepower is constrained by availability of both

ammunition (which is accounted for separately) and personnel.

The fraction  $p$  of firepower of all kinds devoted to flank defense in a given subsector is

$$(4) \quad p = 1 - \frac{d}{d + c\ell}$$

where

$d$  = length of subsector front,

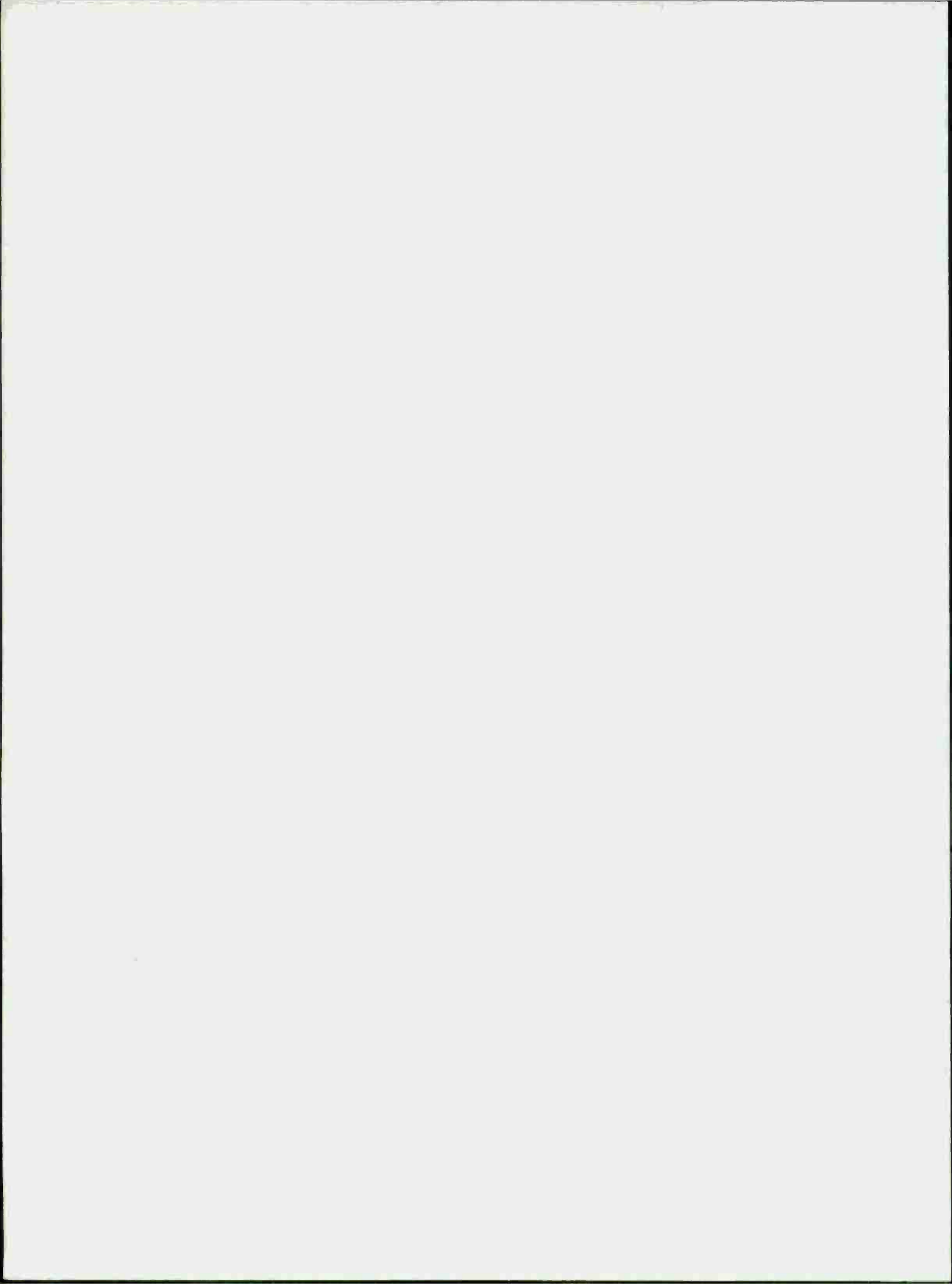
$\ell$  = combined length of exposed subsector flanks (if any),

$c$  = input constant.

Combining the preceding remarks, we note that for each side  $k$ ,

$$ATP(i,k) = \frac{d}{d + c\ell} f\left(\frac{A}{N}\right) \left[ \sum_j ATP(i,j,k) N(j,k) \right]$$

for  $i = 1, 2, 3$ ; other potentials are analogous. Similar representations hold for sources  $i = 4$  (artillery),  $i = 5$  (helicopters) and  $i = 6$  (CAS aircraft). In the latter cases, the supply modifications are weapon-specific, as noted above.



## 5. ATTRITION COMPUTATIONS

### A. GROUND COMBAT ATTRITION

The results of a ground combat engagement are computed from numbers of resources in the combat units engaged, numbers of artillery battalions in support, and numbers of CAS (close air support) aircraft sorties reaching the combat area, using the attrition equations described below.

There are eight distinct engagement types, obtained in the following manner. Each side is in one of four postures: attack, prepared defense, hasty defense, or delay. Combinations of these postures lead to the eight engagement types shown in Table 1. For the rest of the discussion we fix the terrain and engagement types. Of the parameters introduced below, only firepower potentials depend on terrain and engagement type. The computations below are shown only for losses to one side; computations for losses to the other side are, of course, entirely analogous.

Equations (5) through (20) below are valid for all engagements except static engagements; modifications for that case are given following (20).

We first consider losses of tanks. The number  $T_n(d)$  of type  $n$  tanks damaged is given by

$$(5) \quad T_n(d) = T_n \left( 1 - \exp \left[ - \left( k_n \sum_{i=1}^6 ATP(i) \right) / \left( \sum_m T_m \right) \right] \right),$$

where

$T_n$  = number of type  $n$  tanks engaged,

Table 1. ENGAGEMENT TYPE AS A FUNCTION OF POSTURES

Side 2 Posture ↓ Side 1 Posture	Attack	Prepared Defense	Hasty Defense	Delay
Attack	Meeting engagement	Side 1 attack of prepared defense	Side 1 attack of hasty defense	Side 1 advance
Prepared Defense	Side 2 attack of prepared defense	Static engagement	Static engagement	Static engagement
Hasty Defense	Side 2 attack of hasty defense	Static engagement	Static engagement	Static engagement
Delay	Side 2 advance	Static engagement	Static engagement	Static engagement

$T_n(d)$  = number of type n tanks damaged,

$k_n$  = damage coefficient (an input),

ATP(1) = enemy antitank potential from source 1 (see page 11).

This attrition equation is an exponential attrition equation of the type used throughout the Lulejian-I combat model. In particular, we refer the reader to Section 5 of Reference [8] for a detailed discussion of its properties and its relation to other attrition equations such as the binomial equations of Reference [5]. We observe, as is shown in Reference [8], that Equation (5) is an approximation to an equation of Lanchester square form, in terms of both mathematical form and underlying mathematical assumptions. In this physical situation such form may be questionable; a linear law alternative is the binomial equation of Reference [5].

The number  $T_n(k)$  of type n tanks destroyed is given by

$$(6) \quad T_n(k) = d_n T_n(d) ,$$

where

$d_n$  = fraction of damaged type n tanks that are destroyed.

Some of the damaged but not destroyed tanks are abandoned as a consequence of FEBA movement; the number  $T_n(a)$  of abandoned type n tanks is

$$(7) \quad T_n(a) = [T_n(d) - T_n(k)](1 - e^{-bM}) ,$$

where

$b$  = abandonment coefficient (an input),

$M$  = backward FEBA movement.

See Section 6 for details of the computation of  $M$ . Abandonment occurs only if the side being considered loses ground, so that  $M$  is positive. While there is no justification for Equation (7) provided in Reference [9], its form represents an assumption concerning the distribution of tanks as a function of distance from the FEBA. That is, if one chooses a tank at random and denotes by  $D$  its distance from the FEBA, then the form of Equation (7) is an assumption that

$$P\{D \leq t\} = 1 - e^{-bt}$$

for each  $t$ ; that is, the distance is exponentially distributed. This assumption is neither clearly plausible nor patently implausible; it can only be dealt with, however, after it is made explicit. In any case, the errors it induces are not substantial.

Undamaged tanks are subject to mechanical breakdown; the number  $T_n(b)$  of type n tanks that break down is

$$(8) \quad T_n(b) = f_n[T_n - T_n(d)] ,$$

where

$f_n$  = fraction of undamaged type n tanks that break down.

A fraction  $r_n$  of these broken-down tanks are repairable and are sent to a repair pool (from which they return to combat at a later time), as are damaged, but not abandoned or destroyed tanks. The number of  $T_n(r)$  of type n tanks sent to the repair pool is

$$(9) \quad T_n(r) = r_n T_n(b) + [T_n(d) - T_n(k) - T_n(a)] .$$

The following listing displays the final distribution of the initial  $T_n$  type n tanks among (1) undamaged and unbroken, (2) destroyed, (3) abandoned, (4) damaged and sent to repair pool, (5) broken down and sent to repair pool, and (6) broken down and unrepairable:

<u>Category</u>	<u>Number of Type n Tanks</u>
Unscathed	$(T_n - T_n(d))(1 - f_n)$
Destroyed	$T_n(k) = d_n T_n(d)$
Abandoned	$T_n(a) = (T_n(d) - T_n(k))(1 - e^{-bM})$
Damaged - repairable	$T_n(d) - T_n(k) - T_n(a)$
Broken - repairable	$r_n f_n (T_n - T_n(d))$
Broken - unrepairable	$(1 - r_n) f_n (T_n - T_n(d)) .$

The net loss of tanks of type n is  $T_n(k) + T_n(a) + (1 - r_n) f_n (T_n - T_n(d))$ .

Computations of light armor losses are done in an entirely analogous fashion, so we omit details. The developers of the model are to be commended for their explicit representation of damaged but repairable tanks, mechanical breakdowns, abandonments and repairs. These phenomena may be important.

Losses of antitank weapons and mortars are computed using the equation

$$(10) \quad \Delta W_k = f_k (\Delta P / P) W_k ,$$

where

$k$  = weapon type,

$W_k$  = number of type  $k$  weapons engaged,

$\Delta W_k$  = number of type  $k$  weapons destroyed,

$P$  = total personnel in subsector personnel pool  
(see below),

$\Delta P$  = personnel casualties in subsector (Equation (12)  
below),

$f_k$  = fraction of type  $k$  weapons lost per percent  
personnel casualties (an input).

None of the  $\Delta W_k$  weapons lost is salvageable or repairable, so losses are identical with destructions. Equation (10) is a straightforward constant loss rate equation and is probably not grossly in error for fairly small loss rates, although many widely used alternatives exist. Note, however, that weapon losses are computed from personnel casualties. In comparable models such as IDAGAM I and Vector-I, personnel casualties are computed from weapons system losses; the latter procedure appears more plausible in physical terms. Of course, losses to tanks and light armored vehicles are computed directly, as described above.

We next discuss computation and disposition of personnel casualties in the CEM. The number  $\Delta P_1$  of personnel lost from crews of tanks and light armored vehicles is given by

$$(11) \quad \Delta P_1 = \sum_n p_n T_n(d) ,$$

where

$p_n$  = number of personnel lost when a type  $n$  vehicle  
is damaged (an input),

$T_n(d)$  = number of type  $n$  vehicles damaged (see Equation (6)).

Survivors are reassigned to other vehicles or placed in the subsector personnel pool.

All personnel not in vehicles are, for purposes of the model and the following attrition computation, grouped into a single personnel pool. There is one such pool for each subsector in the model; the computation is carried out for each subsector in which combat occurs during the given time period. Personnel casualties  $\Delta P$  in a given subsector are computed using the equation

$$(12) \quad \Delta P = P(1 - \exp[-\frac{k}{N} \sum_1 APP(i)]) ,$$

where

- $P$  = number of personnel in subsector personnel pool,
- $APP(i)$  = enemy antipersonnel firepower from source  $i$  (see Section 4),
- $k$  = personnel vulnerability coefficient (an input that depends on the type of engagement),
- $N$  = number of minisectors in subsector.

Recall that a subsector is simply a group of minisectors, so that the number  $P$  of personnel present in the given subsector is a function of  $N$  as is also, therefore, the number  $\Delta P$  of casualties. Hence Equation (12) can be written more explicitly as

$$(13) \quad \Delta P(N) = P(N)(1 - \exp[-\frac{k}{N} \sum_1 APP(i)]) .$$

Note that fractional casualties are then given by

$$\frac{\Delta P(N)}{P(N)} = (1 - \exp[-\frac{k}{N} \sum_1 APP(i)]) ,$$

and are decreasing in  $N$ , which plausibly represents the effect of forcing the enemy to scatter his fixed firepower over more minisectors. Furthermore,

$$\lim_{N \rightarrow \infty} \frac{\Delta P(N)}{P(N)} = 0 ,$$

which also is both plausible and desirable.

To consider Equation (13) further, let us choose the linear dependence

$$P(N) = p_0 N ,$$

where

$p_0$  = initial number of personnel per minisector,

and is assumed to be constant. The quantity

$$W = \frac{k}{N} \sum_1 APP(i)$$

is the enemy antipersonnel lethality potential directed at each particular minisector, assuming a uniform distribution of subsector potential over the minisectors in the subsector. If one supposes that personnel casualties  $\Delta P_j$  in the  $j^{th}$  minisector are given by the exponential attrition equation

$$\Delta P_j = p_0 (1 - e^{-W})$$

for each  $j$ , then total personnel casualties in the subsector are

$$\Delta P(N) = p_0 N (1 - e^{-W}) ,$$

which is the same as Equation (13) for  $P(N) = p_0 N$ . But an application of the approximation  $e^{-x} \sim 1 - x$  yields

$$(14) \quad \Delta P_j \simeq p_0 k \sum_1 APP(i,j) ,$$

where

$APP(i,j)$  = enemy antipersonnel potential from source  $i$  directed at minisector  $j$  ( $= APP(i)/N$ ).

Equation (14) is clearly a Lanchester linear equation; whether it is appropriate for computing personnel casualties is debatable. Most comparable models employ a square law equation, or some combination of square law and linear law equations, instead.

If  $P$  is not a function of  $N$ , serious difficulties may arise with Equation (12), for in this case attrition is dependent upon the purely artificial variable  $N$ . A change in  $N$ , which alters nothing about the physics of the problem, then leads to a change in the computed value of personnel casualties  $\Delta P$ . For example, when  $P = 1,000$  and  $k \sum_1 \text{APP}(i) = 0.1$ , one obtains the following values of  $\Delta P$  as  $N$  changes:

<u>N</u>	<u><math>\Delta P</math></u>
1	95
2	49
5	20
10	10 .

A point to be emphasized is that since minisectors are not of a fixed geographic width (see Section 2), the variations above reflect no physical reality. Moreover, such variations do arise in the model as different subsectors, containing nearly identical numbers of personnel but varying numbers of minisectors, are treated. The presumed reason for the inclusion of the factor  $N^{-1}$  in the exponential term in Equation (12) is to account for the (real) effect of forcing an enemy to scatter a fixed amount of firepower over a larger geographic area. Within the context of the CEM the method chosen is inappropriate; one alternative would be to replace  $N$  by the geographic front width of the subsector in question.

Another alternative to Equation (12) is the analogous square law equation

$$(15) \quad \Delta P = P(1 - \exp[-\frac{k}{P} \sum_1 \text{APP}(i)]) ,$$

in which all quantities are the same as in Equation (12). That (15) is a square law equation is seen by employing the approximation  $e^{-x} \sim 1 - x$ , which yields

$$\Delta P \cong k \sum_1 \text{APP}(i) .$$

In Equation (15) the personnel strength  $P$  itself serves to represent the decreasing efficiency of increasingly scattered fire. In many circumstances this assumption is not implausible; it is often utilized. Modification of the CEM to incorporate, for example, Equation (15) would be straightforward.

Personnel losses are divided among three classes, namely: killed, wounded, and missing or captured, using the equations

$$(16) \quad \begin{aligned} \Delta P(k) &= f_1 \Delta P \\ \Delta P(w) &= f_2 \Delta P \\ \Delta P(m) &= f_3 \Delta P, \end{aligned}$$

where

$\Delta P(k)[\Delta P(w), \Delta P(m)]$  = number of personnel killed [wounded, missing or captured],

$f_1$  = fraction of losses that are killed,  
 $f_2$  = fraction of losses that are wounded,  
 $f_3$  = fraction of losses that are missing or captured.

Clearly  $f_1 + f_2 + f_3 = 1$ ; the values of the  $f_i$  depend on the type of engagement (see Table 1). The  $\Delta P(k) + \Delta P(m)$  killed and missing or captured are subtracted from personnel totals and dropped from the model.

Of the  $\Delta P(w)$  wounded personnel, the number  $\Delta P(h)$  hospitalized is given by

$$(17) \quad \Delta P(h) = q \Delta P(w),$$

where

$q$  = fraction of wounded personnel taken to theater hospitals;

some of these may be evacuated. The remaining

$$(18) \quad \Delta P(r) = (1-q) \Delta P(w)$$

wounded personnel are treated in battalion aid stations and return to battle during the next division time period.

Hospitalized but unevacuated personnel return to combat after a designated number of time periods.

Noncombat casualties  $\Delta P(n)$  are computed using the equation

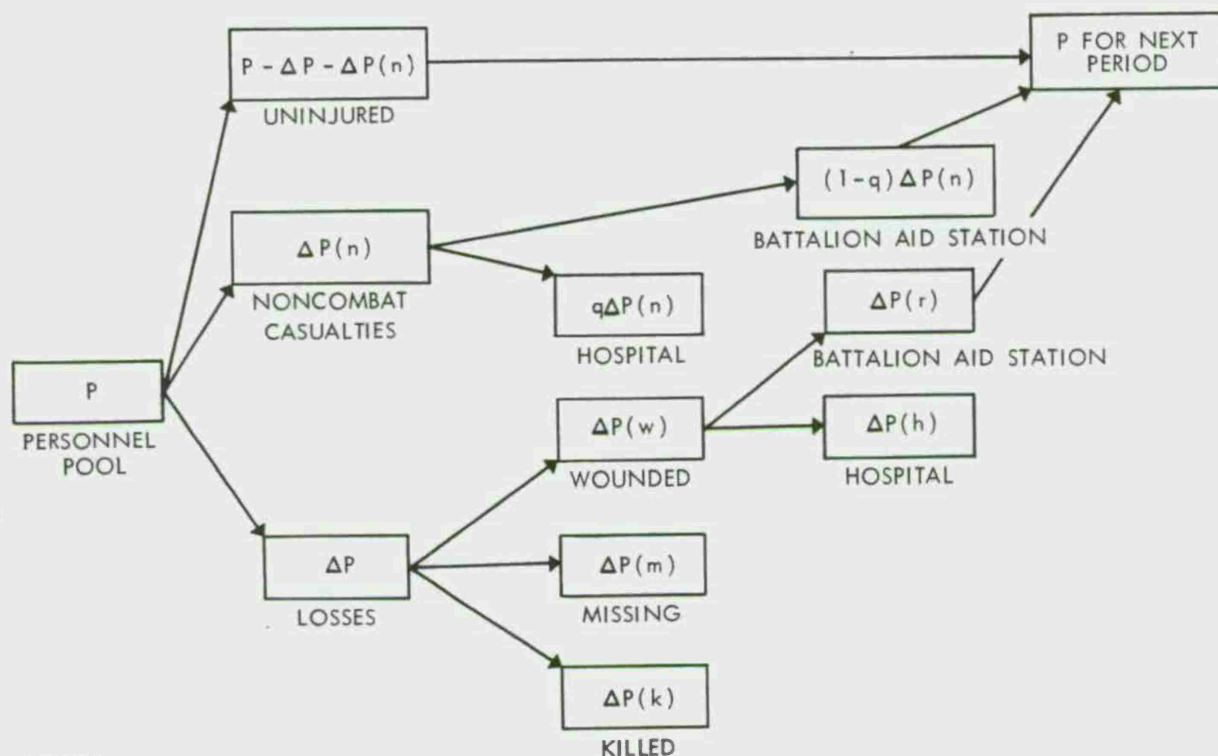
$$(19) \quad \Delta P(n) = u(P - \Delta P),$$

where

$u$  = rate of noncombat casualties (an input).

Noncombat casualties undergo the same disposition as combat casualties (see Equations (16) to (18) above).

Figure 3 summarizes the computation of personnel casualties.



6-8-76-2

Figure 3. COMPUTATION OF PERSONNEL CASUALTIES

Once more, we commend the model developers for their explicit representation of wounded personnel and other potentially important factors not treated in many comparable models.

Helicopter losses are computed in the following manner. The number  $H_n(d)$  of type  $n$  helicopters damaged is given by

$$(20) \quad H_n(d) = S r_n H_n,$$

where

$H_n$  = number of type  $n$  helicopters engaged,

$r_n$  = "expected rate of damage" to type  $n$  helicopters,

$S$  = state of enemy force (see Equation (1)).

The report [9] does not contain an unambiguous definition of  $r_n$ . We believe, however, that  $r_n$  depends on the TOE firepower of the opposition in some manner that attempts to limit losses to acceptable levels. If that dependence is linear and if "rate" in the definition of  $r_n$  means "fraction of helicopters damaged," then (20) is essentially a Lanchester square equation; if "rate" means number of helicopters, then (20) is a Lanchester linear equation.

Numbers of destroyed, abandoned, broken down, and repairable helicopters are computed in the same way as the corresponding quantities for armored vehicles; the reader is referred to pages 16-17 for details.

The preceding calculations are applicable to all types of engagements except the static type (see Table 1). For static engagements the methodology above is modified as follows:

- (1) Losses can result only from direct support artillery and CAS aircraft. The antitank potential of Equation (5), the analogous anti-light armor potential, and the antipersonnel potential of Equation (12) are modified to include only firepower contributions from these sources and are then used as previously described.

- (2) No abandonments of tanks, light armored vehicles or helicopters occur since there is (by definition of a static engagement) no movement of the FEBA.

Except for these changes, computations for a static engagement are carried out in the same manner as those for other types of engagements.

Losses to personnel and armor in reserve units can arise only from artillery in the general support (GS) role and from CAS aircraft. With suitably modified antitank and anti-personnel potentials (including only contributions from these sources), Equations (5) and (12) apply, as does the analogous equation for losses of light armored vehicles. The equation for personnel casualties contains a reserve vulnerability coefficient together with an adjustment for the fraction of GS fire directed at the unit under consideration. There is no abandonment of armored vehicles.

In addition to causing casualties, enemy artillery in the GS role can neutralize a fraction of friendly DS artillery; this neutralization is computed before any other attrition calculations so that only unneutralized DS artillery can cause or incur destruction. All numbers of artillery used in the preceding attrition calculations are numbers of unneutralized artillery. The neutralized fraction  $f$  is given by

$$(21) \quad f = 1 - e^{-k(APP)/M},$$

where

$M$  = number of friendly DS artillery battalions,

$APP$  = enemy antipersonnel potential from counterbattery fire,

$k$  = neutralization vulnerability coefficient (an input).

Use of antipersonnel potential in Equation (21) implies that neutralization occurs as a result of casualties to artillery crew personnel, which is a plausible assumption.

Losses  $\Delta P_a$  to personnel in direct support (DS) artillery units can arise only from counterbattery fire and are computed using the equation

$$(22) \quad \Delta P_a = P_a (1 - \exp[-\frac{\ell}{M} APP]) ,$$

where

$P_a$  = number of personnel in DS artillery units,

$\ell$  = personnel vulnerability to counterbattery fire,

APP = enemy antipersonnel potential from counterbattery fire,

M = number of friendly DS battalions.

Note that  $P_a$  is seemingly proportional to M (see the discussion following Equation (12)). Losses  $\Delta C_n$  of type n cannon are

$$(23) \quad \Delta C_n = C_n \left( 1 - \exp \left[ - \frac{k_1 ATP + k_2 ALP}{\sum_m C_m} \right] \right) ,$$

where

$C_n$  = number of (unneutralized) type n cannon engaged,

$k_1$  = vulnerability of cannon to enemy counterbattery antitank firepower,

ATP = enemy counterbattery antitank potential,

$k_2$  = vulnerability of cannon to enemy counterbattery anti-light armor firepower,

ALP = enemy counterbattery anti-light armor potential.

All hits, in this case, cause destruction. Equation (23) is an exponential equation of the kind used throughout the Lulejian-I model; the reader is referred to Reference [8] for details concerning it and for suggested alternatives.

## B. AIR COMBAT ATTRITION

The CEM represents only two types of tactical aircraft: a notional air defense fighter (ADF) and a notional tactical

fighter (TF); some of the TF aircraft function as sweep fighters (SF) in defense of TF aircraft performing air-to-ground missions. Missions performed by tactical aircraft are:

- INT = air defense intercept (ADF only),
- CA = counter air, namely destruction of enemy aircraft at bases, base facilities and SAM sites (TF only),
- CAS = close-air support of ground combat (TF only),
- R/I = reconnaissance and interdiction (TF only).

No special-purpose aircraft are included in the model. Allocation of TF aircraft to missions is based on aircraft loss rates and on the state of the ground combat and is described in more detail in Section 8.

Each side may have two kinds of air bases--primary air bases, on which all ADF are assumed to be based, are within range of enemy aircraft; while secondary air bases are immune to attack by enemy aircraft. TF aircraft may be on either type of base at the discretion of the model user; however, basing must be consistent with aircraft range and mission choice.

The air battle--one half of which is described below--proceeds in the following manner:

- (1) Enemy aircraft attempting penetration are (possibly) detected by a central control and warning system (the only detection process model in the air combat portion of the CEM),
- (2) ADF aircraft are assigned to seek to engage penetrators,
- (3) ADF aircraft attack and attempt to destroy penetrators,
- (4) Penetrators that have survived attacks by ADF aircraft seek, in return, to attack and destroy the ADF aircraft,
- (5) Unengaged TF aircraft attack targets on the ground.

Penetrating aircraft are TF aircraft on CA and R/I missions and SF aircraft serving as escorts of TF aircraft. Apparently no air-to-air combat occurs involving aircraft on the CAS mission. These aircraft are, of course, vulnerable to ground-based defenses; see Equation (36).

The reasoning underlying inclusion of part (4) of the air combat process is unclear even if it is assumed, as in the CEM, that engaged TF aircraft jettison their ordnance at once. It seems to us that even under this assumption, the main goal of a TF aircraft, if attacked, is to elude the attacker and return safely to the air base and not to engage in retaliatory attacks. Especially if ADF aircraft have superior armaments or capabilities for air-to-air combat, TF aircraft will try only to escape unscathed. This may not be true of SF aircraft.

We next describe in more detail the mathematical computations used to represent air combat in the model. The fraction  $f$  of penetrating TF and SF aircraft *not engaged* is given by

$$(24) \quad f = (1-p)^{[A/(T_c + S_c + T_r + S_r)]},$$

where

$T_c$  = number of TF aircraft attempting penetration on CA mission,

$S_c$  = number of SF aircraft attempting penetration on CA mission,

$T_r$  = number of TF aircraft attempting penetration on R/I mission,

$S_r$  = number of SF aircraft attempting penetration on R/I mission,

$A$  = number of enemy ADF aircraft present,

$p$  = probability of intercept (an input).

According to Reference [9], the intercept probability  $p$  includes probabilities of successful detection of a penetrator by the centralized detection system, availability of an interceptor aircraft, and success of the attempted intercept.

We know of no rigorous derivation of Equation (24); it is superficially similar to the exponential equations used in the Lulejian-I model. If  $p$ , however, is to include all the factors

indicated, we fail to see the reason for the exponent in Equation (24); the unengaged fraction  $f$  is simply  $(1-p)$ . Several alternatives to Equation (24) are available; some of these are the Markov barrier penetration processes described in Reference [2]. Another alternative, in view of the assumption (mentioned below) that exactly one interceptor is assigned to attempt to engage each detected penetrator, would be the equation

$$(25) \quad f = 1 - p' \cdot \min \left\{ 1, \frac{A}{p'(T_c + S_c + T_r + S_r)} \right\} q ,$$

where

$p'$  = probability that a penetrator is detected,

$q$  = probability that an attempted intercept is successful,

and where  $A$ ,  $T_c$ ,  $S_c$ ,  $T_r$ ,  $S_r$  are as above. Here  $\min \{1, A/p'(T_c + S_c + T_r + S_r)\}$  is the probability that there is an interceptor available given that a penetrator is detected. In our opinion, Equation (24) needs to be, and can be, improved.

The  $fT_c$  and  $fT_r$  unengaged TF aircraft assigned to CA and R/I missions, respectively, proceed toward their targets; unengaged SF aircraft return to their bases without further incident.

It is assumed that engagements between ADF aircraft and detected penetrators occur only on a one-on-one basis and that ADF aircraft always attack first. The number  $\Delta T(1)$  of TF aircraft destroyed is thus

$$(26) \quad \Delta T(1) = (1-f)(T_c + T_r)q ,$$

where

$q$  = probability that an ADF aircraft kills a TF aircraft in a one-on-one engagement.

Under the assumption that surviving TF aircraft counterattack, the number  $\Delta A$  of ADF aircraft destroyed is then given by

$$(27) \quad \Delta A = (1-f)(1-q)(T_c + T_r)q' ,$$

where

$q'$  = probability that a TF aircraft kills an ADF aircraft in a one-on-one engagement, given that the latter has not succeeded in killing the former.

As mentioned before, we believe that there are good reasons for omitting entirely the interaction described by Equation (27). This can easily be accomplished by a judicious choice of parameters (i.e.,  $q' = 0$ ).

Losses of SF aircraft and kills of ADF aircraft by surviving SF aircraft are calculated analogously; here the second interaction is more plausible than for TF aircraft.

Next we consider effects of ground-to-air defenses. The number of  $\Delta T_c(2)$  of TF aircraft on the CA mission killed by ground-to-air defenses (i.e., SAMs and AAA) is given by

$$(28) \quad \Delta T_c(2) = (fT_c)(ke)(gN) ,$$

where

$fT_c$  = number of TF aircraft in CA mission not engaged by ADF aircraft (see Equation (24)),

$e$  = probability of engagement by each air defense unit,

$k$  = probability of kill given engagement by one air defense unit,

$g$  = fraction of air defense units encountered in CA mission,

$N$  = "number" of air defense units present in the subsector.

No distinction is made in Equation (28) between AAA and the more effective SAMs, except that the number  $N$  is computed from the numbers of the two different systems present using weighting factors relating the two types of defenses. This, we believe, is an inadequate method of representing the important

and substantial differences between SAMs and AAA. Nearly every comparable model treats the problem more carefully and plausibly than the CEM (see References [1], [3], [7], and [8]). Equation (28) is a Lanchester linear equation and--in view of the underlying physical assumptions set forth in References [4] and [6]--seems appropriate in this context.

The number  $\Delta T_r(2)$  of TF aircraft in the R/I role that are destroyed by ground-based defenses is

$$(29) \quad \Delta T_r(2) = (fT_r)(ke)(g'N) ,$$

where

$g'$  = fraction of air defense units encountered in R/I mission,

and where all other quantities are as in Equations (24) and (28).

Destruction of enemy aircraft on air bases by TF aircraft on the CA mission is computed next. The number of shelters on an air base is fixed and aircraft are sheltered (proportionally to the numbers present and without regard to type) to the extent possible. A hit on a shelter destroys the contents but not, it is assumed in this model, the shelter itself. Comparable models such as IDAGAM I adopt the seemingly more realistic viewpoint that shelters can be destroyed.

The fraction  $h_i$  of type  $i$  enemy aircraft assigned to a given air base that are at risk during an attack is given by

$$(30) \quad h_i = (1 - \frac{s_i t_i}{\ell})(1 - q_i) ,$$

where

$i$  = aircraft type ( $1 = \text{TF}$ ,  $2 = \text{ADF}$ ),  
 $s_i$  = sortie rate for type  $i$  aircraft,  
 $t_i$  = sortie duration for type  $i$  aircraft,  
 $\ell$  = length of vulnerable period of air base to attack,

$q_1$  = probability that an aircraft of type 1 can take off upon warning of imminent attack.

Presumably  $s_1 t_1 \leq 1$ .

The number  $B$  of enemy aircraft at risk is thus given by

$$(31) \quad B = h_1 [B^{(1)} - \Delta B^{(1)}] + h_2 [B^{(2)} - \Delta B^{(2)}] ,$$

where

$B^{(1)}$  = number of type 1 enemy aircraft assigned to the air base at the start of the time period,

$\Delta B^{(1)}$  = losses of enemy TF aircraft in CA and R/I missions,

$\Delta B^{(2)}$  = losses of enemy ADF aircraft in INT mission.

The number of sheltered aircraft is

$$(32) \quad B_s = \min \{S, B\} ,$$

where

$S$  = number of shelters on the air base,

and the number of unsheltered aircraft is

$$(33) \quad B_u = B - B_s .$$

The number  $\Delta B$  of enemy aircraft destroyed by TF aircraft on the CA mission that attack the air base is therefore

$$(34) \quad \Delta B = B_s \left[ 1 - (1-p_s)^{(1-f)\tilde{T}_c/B} \right] \\ + B_u \left[ 1 - (1-p_u)^{(1-f)\tilde{T}_c/B} \right] ,$$

where

$p_s [p_u]$  = probability of kill of one sheltered [unsheltered] aircraft by one attacking aircraft,

$f$  = fraction of CA sorties assigned to SAMs (see Equation (35), below),

$\tilde{T}_c$  = number of attacking aircraft on CA mission that have survived all defenses and are assigned to attack the air base,

and where  $B_s$ ,  $B_u$  are computed in Equations (32) and (33). Equation (34) is of Lanchester square form (see Reference [8]) and seems, on the basis of underlying assumptions, to be plausible at least in regard to unsheltered aircraft. Unsheltered aircraft are vulnerable to "area fire" weapons and such interactions may be represented by a Lanchester square equation (see Reference [4]). Sheltered aircraft, on the other hand, seem to represent a detection problem which is better represented by a Lanchester linear equation (see References [4] and [5]). Some models comparable to the CEM are able to account for this phenomenon.

The number  $\Delta S$  of enemy SAM sites destroyed by TF aircraft in the CA role is given by

$$(35) \quad \Delta S = qf\tilde{T}_c ,$$

where

$q$  = probability that an aircraft on CA mission destroys a SAM site,

and where  $f$ ,  $\tilde{T}_c$  are as in Equation (34). This equation is of Lanchester square form, which seems appropriate.

Aircraft on the R/I mission cause no losses to enemy weapons or personnel and affect the course of the ground combat only by decreasing certain resource arrival rates.

Finally, losses  $\Delta A$  to TF aircraft on the CAS mission are given by

$$(36) \quad \Delta A = \frac{S \cdot t \cdot U}{w} ,$$

where

$S$  = number of squadrons of TF aircraft assigned to the CAS mission,

$t$  = number of TF aircraft lost per squadron, per enemy air defense unit,

$U$  = number of enemy air defense units present in subsector,

$w$  = width of subsector in minisectors.

The computation is performed once for each subsector;  $U$  and  $w$  are essentially proportional; the equation is hence of linear law form. There exists no clear reason for the sudden change to accounting for aircraft squadrons rather than individual aircraft.

Attrition caused by CAS aircraft is computed by means of firepower potentials, as previously described.

We conclude this section with some general comments concerning the attrition methodology in the CEM.

The most serious criticism concerns the predicating of all attrition computations on firepower potentials. Difficulties with this method of attrition computation (e.g., scaling problems, compatibility, linearity, lack of representation of interactions, difficulty in obtaining input data, etc.) are well-known and need not be discussed in detail here. The CEM suffers from all such difficulties. A few alternatives to firepower potential-based computations exist in implemented form; these alternatives are in the IDAGAM I and the Vector-I models (see References [1], [3] and [7]). Other alternatives remain at a theoretical stage of development (see References [2], [4], [5] and [6]). So far as we can tell, no attempt is made in the CEM to mitigate any of the previously mentioned difficulties with firepower potential-based attrition computations. This is, in our opinion, a serious shortcoming of the model.

While none of the attrition equations is, in the context of firepower potentials, patently wrong or inappropriate, more plausible alternatives seem to exist for many of them. In particular, computation of personnel casualties, computation of the fraction of engaged penetrator aircraft, and computation of losses of CAS aircraft seem to require improvement. Each attrition equation, with the possible exception of that used to compute engagement rates in the air battle, has an (more-or-

less) appropriate grounding in terms of some known set of assumptions and a derivation therefrom. Use of unnecessary exponential approximations should be avoided, however.

The principal personnel attrition equation--Equation (12) above--contains an artificial factor that is intended to account for a real physical phenomenon, but does not do so properly. That is, scattering of firepower is erroneously represented in terms of the number of minisectors in the subsector under consideration. This number of minisectors is an artificial number, internal to the model, whose value then affects, possibly rather substantially, the computed level of personnel attrition. The difficulty can be mitigated within the context of Equation (12) by using instead a geographic parameter such as the width of the subsector front or subsector personnel strength, which yields (in addition) a square law equation. This difficulty does not appear to introduce a bias in favor of one side, so certain comparative analyses may not be negated by it; one cannot be sure of this, however.

As do all iterative deterministic simulation models that attempt to represent stochastic phenomena, the CEM repeatedly replaces random variables by their expectations. It is unlikely that the errors so created are significantly greater than either those appearing from the same source in other models or those appearing from other sources in this model.

Only one logical flaw seems to exist in the structure of the attrition processes and it is less a logical flaw than a difference in interpretation of the physics of combat: this is the inclusion of counterattacks on interceptor aircraft by engaged but unkilld penetrators. We do not believe that such attacks occur but that, instead, penetrators are content with having escaped. Computation of losses of hand-held weapons from losses of personnel (rather than losses of personnel from losses of weapons systems) also seems questionable to us.

The role of SF as escorts or defenders of TF aircraft is unclear. Except insofar as they destroy some of the ADF which engage them, the SF do not affect the attrition of TF. A secondary effect occurs through the engaged fraction of aircraft computed in Equation (24); if  $S_c$  and  $S_r$  are decreased, the fraction of TF aircraft engaged by interceptors increases. In this context the role of SF seems to be essentially only diversionary.

Explicit inclusion of damaged but renewable resources (e.g., wounded personnel and repairable armored vehicles) is a useful and positive aspect of the model. It seems that especially in a protracted conflict, the side with the greater ability to renew itself may well prove victorious. On the other hand, the victorious side may be determined less by its ability to destroy enemy resources than by its ability to damage them and keep them out of action long enough to allow an insurmountable advantage in position to be obtained. Possibly both of these phenomena are important; the CEM has the capability (not so explicitly present in some comparable models) to investigate such questions.

Like all other comparable models, the CEM fails entirely to represent synergistic effects among different weapon types on the same side and any effects, other than direct attrition, among weapons on opposite sides.



## 6. FEBA MOVEMENT COMPUTATION

In this section we describe the CEM methodology for computation of FEBA movement; the notation used for various potentials is that of Section 4. FEBA movement  $M$  is a discrete function of a force ratio  $R$  in which contributing firepower potentials are weighted in terms of targets as well as contributing weapons. As are other assessment computations, the FEBA movement computation is performed once per division time period for each subsector in the theater. We shall now describe the computations in more detail.

The basic computation yields for each minisector a FEBA movement  $M$  as a piecewise constant function of the minisector attacker-to-defender force ratio  $R$ :

$$(37) \quad M = \begin{cases} m_1 & \text{if } R \leq a_1 \\ m_2 & \text{if } a_1 < R \leq a_2 \\ m_3 & \text{if } a_2 < R \leq a_3 \\ m_4 & \text{if } a_3 < R \leq a_4 \\ m_5 & \text{if } a_4 < R \end{cases}$$

where

$m_1$  = possible FEBA movements (input values with  $m_1 < m_2 < m_3 < m_4 < m_5$ ),

$a_1$  = movement thresholds (input values with  $0 < a_1 < a_2 < a_3 < a_4$ ).

Presumably, but apparently not necessarily, one of the  $m_1$  is zero. All the  $m_1$  must be multiples of the basic unit of terrain battlefield depth, namely 0.1 kilometer (cf. Section 2). The movement values  $m_1$  and thresholds  $a_1$  depend on the type of terrain, type of engagement and possible presence of a major obstacle. FEBA movement can occur only for engagements other than static engagements; the attacker in a meeting engagement is taken to be the Blue side. This assumption is probably harmless. Its effect can easily be ascertained by modifying the model to make Red the attacker in meeting engagements, repeating previous runs of the model, and comparing the results.

To compute FEBA movement as a function of force ratio, most models comparable to the CEM use a piecewise linear function rather than a piecewise constant function. The former methodology appears to be both more general and more realistic, requires essentially no more computer storage, and involves only slightly more computation (a linear interpolation). The CEM could easily be modified to incorporate this improvement.

The force ratio  $R$  includes firepower contributions arising from maneuver units in brigades, artillery battalions, cavalry units and CAS aircraft. All firepower modifications described in Section 4 are applied to potentials used in the computations described here. There is, however, in this case (but *not* for any attrition computations) an additional modification that weights firepower potentials in terms of potentials generated by the targets present. As in Section 4, for each  $i$  and  $k$  let  $ATP(i,k)$  ( $ALP(i,k)$ ,  $APP(i,k)$ , respectively) be the antitank potential (anti-light armor potential, antipersonnel potential, respectively) of side  $k$  arising from source  $i$ . These quantities are assumed to reflect the modifications for

supply shortages and flank exposure discussed in Section 4. The *modified antitank potential* of Side 1 (for example) is given by

$$(38) \quad \text{MATP}(1) = \left( \sum_{i=1}^6 \text{ATP}(i,1) \right) \times \frac{\text{ATP}(1,2) + \text{ALP}(1,2) + \text{APP}(1,2)}{\left[ \sum_{\ell=1}^3 (\text{ATP}(\ell,2) + \text{ALP}(\ell,2) + \text{APP}(\ell,2)) \right]} .$$

This weighting scheme may be explained as follows. The value  $\left[ \sum \text{ATP}(i,1) \right]$  is the total antitank potential of Side 1. Recalling that source  $i = 1$  corresponds to tanks, the value

$$\text{ATP}(1,2) + \text{ALP}(1,2) + \text{APP}(1,2)$$

is the total potential of Side 2 which arises from tanks, while

$$\sum_{\ell=1}^3 [\text{ATP}(\ell,2) + \text{ALP}(\ell,2) + \text{APP}(\ell,2)]$$

is the total potential of Side 2 arising from the sources, namely tanks, light armored vehicles and personnel, *against which the Side 1 potentials are directed*. Consequently, the ratio term on the right-hand side of Equation (38) is the fraction of firepower potential on Side 2 that is generated by tanks. Here, "fraction" means relative to potential arising from sources against which potentials of Side 1 are directed.

Similar computations yield the analogous quantities

MALP(1) = modified anti-light armor potential of Side 1,

MAPP(1) = modified antipersonnel potential of Side 1,

MATP(2) = modified antitank potential of Side 2,

MALP(2) = modified anti-light armor potential of Side 2,

MAPP(2) = modified antipersonnel potential of Side 2.

The force ratio R is then given (assuming, for illustrative purposes, that Side 1 is the attacker in the subsector) by

$$(39) \quad R = \frac{MATP(1) + MALP(1) + MAPP(1)}{MATP(2) + MALP(2) + MAPP(2)} .$$

The modification effected by Equation (38) is best interpreted, we feel, as an allocation of fire that is independent of the type of shooting weapon. If every weapon of class i on Side 1 devoted a fraction of its effort f(i,1) given by

$$(40) \quad f(j,1) = \frac{ATP(j,2) + ALP(j,2) + APP(j,2)}{\left[ \sum_{\ell=1}^3 (ATP(\ell,2) + ALP(\ell,2) + APP(\ell,2)) \right]} ,$$

to attempts to destroy enemy sources of class j (for j=1,2,3), then Equation (38) reflects such an allocation of fire. In this interpretation, a given shooting weapon on Side 1 will then produce, during a given time period, f(1,1) of its maximum possible ATP, f(2,1) of its maximum possible ALP and f(3,1) of its maximum possible APP. It is to be noted, however, that the allocation is the same for all shooting weapons, which is not plausible, and that such allocation is nowhere reflected in attrition computations.

Subsector FEBA movement is obtained by averaging, with respect to relative geographic frontages, the previously computed minisector FEBA movements for the minisectors in the given subsector.

The following additional facts should also be noted. A subsector attacker with two exposed flanks cannot advance, even if the opposition is exceedingly weak. Flank length constraints of the usual sort are used to effect geographic retraction of the FEBA. Major obstacles cannot be crossed without a separate engagement requiring a full division time period; movements that would result in such a crossing are truncated.

## 7. DECISION-MAKING PROCESSES

A distinguishing feature of the CEM is a complex and intricate representation of hierarchical decision making during the course of a campaign. Decision-making processes at theater, army, corps, and division levels are represented; the higher the level (as noted in Section 3) the less frequently decisions can be changed. At time periods when decisions are taken on more than one level, decisions are made in order of decreasing level. Consequently, decisions at each echelon are always constrained by and made in light of decisions at higher levels. The principal decisions are:

- (1) mission selection, in which the general rule is to choose the most aggressive attainable mission;
- (2) allocation of fire support, in which the general rule is to support strength on offense and weakness on defense;
- (3) commitment and reconstitution of reserves, in which the general rule is also to support offensive strength and defensive weakness, but only if necessary.

The three decisions are not independent of one another. For example, the possibility of reserve reconstitution depends on mission selection.

The principal basis on which these decisions are made is the comparison of unit strengths (determined from status files) with an estimated strength of the enemy forces to be engaged. Force ratios are used for such comparisons. The "strength" of a unit can depend on the mission under consideration. Estimated enemy strength  $E_n$  at time  $n$  is (notionally) given by

$$(41) \quad E_n = aA_{n-1} + bA_{n-2} ,$$

where

$A_k$  = actual strength of opposing enemy forces at time  $k$ ,  
 $a, b$  = intelligence parameters (inputs).

The purpose of the estimated strength is to give a (highly stylized) representation of the effect of timeliness of intelligence reports. The weighting factors  $a$  and  $b$  may be different for the two sides and need not sum to 1. When  $a + b = 1$ , Equation (41) represents a weighted delay in the information available concerning enemy strengths. Additional implicit assumptions are involved, however, when  $a + b \neq 1$ ; for example,  $a + b > 1$  can be interpreted as a belief that the enemy is increasing its force strength by means of reinforcement or as a very conservative estimate of the situation. Again, we emphasize the importance and desirability of making assumptions explicit so that they may be dealt with in a meaningful manner. For the Blue side only there exists the option to replace Equation (41) by

$$(42) \quad E_n = a'A_n + b'A_{n-1}.$$

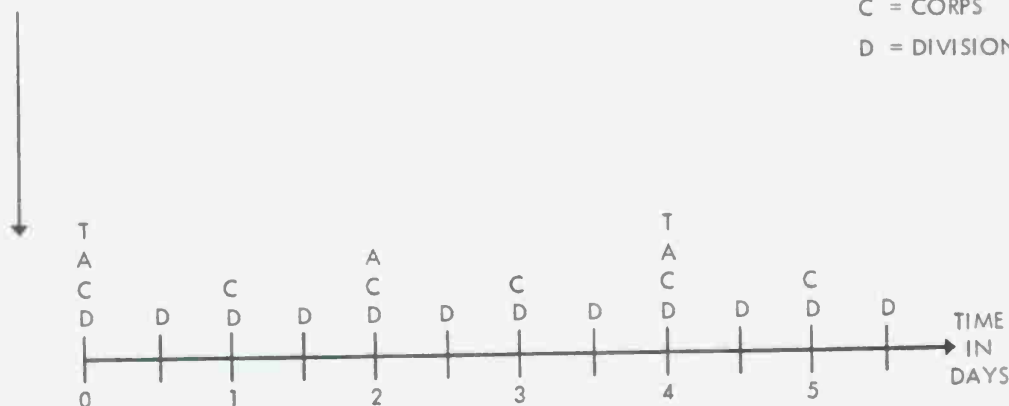
Since (42) depends on  $A_n$ , The Red side must already have made the decisions leading to this quantity, hence, (42) can be used only by one side. This particular asymmetry is unavoidable.

A disadvantage of this scheme is the computer storage required. As discussed in detail in Section 3, resource accounting in the CEM is by means of unit status files, so that use of Equation (41) or (42) requires that three status files be maintained for each unit. This statement is not quite true in that files corresponding to past time periods are less detailed than the current file; nonetheless the effect on storage requirements is still substantial.

We now proceed to a more detailed discussion of the decision-making processes in the CEM. These processes are structured into cycles at the theater, army, corps, and division levels. Each cycle at one level contains subcycles corresponding to the next lower level in the same manner as the various time periods are multiples of each other (see Section 3). Decisions taken at each echelon can be changed only at the beginning of a cycle of that echelon; a sample situation is shown schematically in Figure 4. In this example, the cycle lengths are half a day for divisions, one day for corps, two days for armies, and four days for the theater. Engagements and assessments occur once each division cycle.

ORDER IN WHICH  
DECISIONS  
ARE MADE

T = THEATER  
A = ARMY  
C = CORPS  
D = DIVISION



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Figure 4. DECISION CYCLES IN CEM

In the *theater cycle* the following are accomplished:

Assignment of reinforcement artillery battalions to armies. Numbers of assigned battalions are in proportion to numbers of divisions in armies.

Assignment of CAS sorties to armies. Sorties are assigned in proportion to numbers of divisions in armies.

Determination of logistic resupply above the division level. This function includes constitution of theater supply and personnel pools and establishment of maintenance and transportation rates and policies.

In the *army cycle* decisions are made effecting the following:

Assignment of reinforcing divisions to corps. According to user option, either all corps are eligible or only those corps in a designated army are eligible to receive a given reinforcing division. When the assignment is made, corps on delay receive first priority, followed by corps on attack (in order of decreasing force ratio) and then by corps on defense (in order of increasing force ratio). If assignment of the new division results in a corps of five or fewer divisions, the previous reserve division is placed on line and the new division becomes the corps reserve. If a six division corps results, two new three-division corps are created. A resultant six-corps army is similarly divided.

Force ratio estimation. For purposes of mission selection, each Blue and Red army computes the ratio of its strength to the estimated strength of the enemy forces facing it across the FEBA. The "strength" to be computed depends on the mission under consideration in that only friendly maneuver units capable of that mission are counted; friendly strength, moreover, excludes the army reserve (which is one corps) or if there is no army reserve, the weakest on-line corps in the army. The force ratio considered for mission  $i$  is of the form

$$(43) \quad R_A(i) = A(i)/E ,$$

where

$A(i)$  = army firepower potential for mission  $i$ ,

$E$  = estimated enemy firepower potential (see Equations (41), (42)).

The army firepower potential  $A(i)$  for mission  $i$  is given by

$$(44) \quad A(i) = \sum_m F(m)S(m) + gH + \sum_l K(l) .$$

Here the first summation is over maneuver units in the army that are capable of undertaking mission  $i$ , the second summation is over artillery battalions and

$F(m)$  = TOE firepower potential of  $m^{\text{th}}$  maneuver unit,

$S(m)$  = state of  $m^{\text{th}}$  maneuver unit,

$H$  = number of helicopters in corps and subsidiary units,

$g$  = firepower potential of one helicopter,

$K(l)$  = firepower potential of  $l^{\text{th}}$  artillery battalion.

Presumably all potentials are lumped together, with the usual computability difficulties (which also arise in the FEBA movement calculation described in Section 6). None of the modifications discussed in Section 4 is operative here.

**Mission selection.** Once the force ratios

$R_A(a)$  = force ratio for attack mission,

$R_A(d)$  = force ratio for defense mission,

and

$R_A(w)$  = force ratio for delay (withdrawal) mission,

are computed using Equations (43) and (44), mission determination is as follows:

- (1) If  $R_A(a)$  exceeds an upper attack threshold  $a^+$ , an attack mission with reserve is selected (if no reserve previously existed, the weakest on-line corps is withdrawn and the army reserve reconstituted);
- (2) If  $a^- \leq R_A(a) < a^+$ , where  $a^-$  is a lower attack threshold, an attack mission is chosen and the army reserve committed (although the reserve need not go on line immediately);

- (3) If  $R_A(a) < a^-$ , no attack can be undertaken. The value of  $R_A(d)$  is then considered. If  $R_A(d) \geq d^+$ , where  $d^+$  is an upper defense threshold, the mission is defense with reserve (if no reserve previously existed, the weakest on-line corps is withdrawn).
- (4) If  $d^- \leq R_A(d) < d^+$ , where  $d^-$  is a lower defense threshold, the army mission is defense without reserve.
- (5) If  $R_A(d) < d^-$ , the mission is delay with reserve or delay without reserve according to whether  $R_A(w)$  exceeds a withdrawal threshold  $w$  or not.

A reconstituted reserve leaves the front immediately.

**Reserve commitment or reconstitution.** This decision is made in conjunction with mission selection as described above.

**Corps boundary adjustments.** As a more or less unique feature, the CEM permits adaptive adjustments of force boundaries in an attempt to frustrate enemy attempts to gain local superiority. These adjustments are permitted only when the army mission is defense or delay and allow shifting of inter-corps boundaries within the corps. For details we refer the reader to Reference [9].

**Allocation of artillery to corps.** Allocation of artillery battalions assigned directly to the army (rather than to a subordinate division) is done in a manner that supports strength on offense and weakness on defense. For the  $i^{\text{th}}$  corps in the army let

$$(45) \quad \tilde{R}_C(i) = \begin{cases} R_C(i) & \text{if army mission is attack} \\ 1/R_C(i) & \text{if army mission is defense or delay,} \end{cases}$$

where  $R_C(i)$  is the corps force ratio, which is computed in the same manner as the army force ratio. The fraction of artillery support allocated to the  $i^{\text{th}}$  corps is then

$$(46) \quad f(i) = \tilde{R}_C(i) / \sum_l \tilde{R}_C(l) ,$$

where the summation is over all corps in the army. Artillery support is thus allocated to constituent corps in proportion to the  $\tilde{R}_C(i)$ . If the army mission is attack, then  $\tilde{R}_C(i) = R_C(i)$  so the stronger corps (in terms of strengths relative to the enemy forces opposing them) receive relatively more support. If, on the other hand, the army mission is defense or delay, then  $\tilde{R}_C(i) = 1/R_C(i)$  and constituent corps that are relatively weaker (compared to enemy forces facing them) receive more support. This apportionment seems to us a sensible and worthwhile method of allocation; it deserves more frequent use.

Allocation of CAS sorties to corps. CAS sorties allocated to the army are allocated among the corps comprising the army using the same methodology as for allocation of artillery battalions.

In the *corps cycle*, essentially the same decisions are made by each corps as are made by armies in the army cycle--only allocations are now among component divisions. Indeed, an accurate description of the corps decision making, with one exception, can be obtained by substituting "corps" for "army" and "division" for "corps" in the preceding description. We therefore omit further details. The exception is that for the Red side only there exists the user option to permit personnel and weapon replacements only to off-line Red divisions that are in the process of being rebuilt. That is, no personnel or weapon replacements are allotted to on-line Red divisions. This exception gives the CEM some capability to model the difference between unit and individual replacement policies. Red divisions in reserve may retain, also at user option, organic artillery, rather than giving up such artillery to the parent corps until the division comes on line.

The *division cycle* is basic in the CEM in that engagements occur once during each division cycle. Each Blue division

undertakes estimations and decisions leading to the following choices and allocations.

Mission selection for each component brigade. For each brigade and potential mission an enemy strength opposing that mission is estimated. The actual friendly strength and estimated enemy strength are used as inputs to the FEBA movement function (see Section 6) to obtain an estimated engagement outcome. The engagement outcome estimate is initially based on the brigade's being allotted one-third of the division's available cavalry and artillery support, but no CAS support. The estimation is then repeated to consider the effects of more DS artillery, the assignment of some GS artillery to the DS role, and the commitment or reconstitution of the division reserve. Then by using criteria that are not explained in Reference [9], a mission that achieves the "most favorable outcome with the least support" is selected for each brigade. Once this is done, artillery of an uncommitted reserve brigade is allocated among on-line brigades, incompatible decisions (e.g., attack and delay missions by adjacent brigades) are mitigated by adjustment of brigade frontages, and cavalry unit assignments are adjusted.

Tactical decisions at the division level by the Red side consist only of mission selection for each division and of allocation of artillery between DS and GS roles.

Blue and Red personnel and supply replenishments are carried out once each division cycle, after engagement effects have been computed and assessed. Resupply to on-line units is proportional to needs, with excess stocks retained in theater-level pools. Weapons and supplies are immediately available at full effectiveness to on-line divisions, but newly arrived personnel must be assimilated over a period of time. Withdrawal of Red divisions to which individual replacements are prohibited occurs when the division state falls below a

prescribed threshold; eligibility for return obtains when the state of a rebuilt division exceeds a different threshold and a specified time period has elapsed. Artillery ammunition, on both sides, is accounted for and resupplied independently of other classes of supplies.

Our overall impression of the decision-making structure represented in the CEM is mixed. The explicitness and detail of the structure are commendable; at least one knows what one is dealing with. An extreme form of an argument we have advanced in other places is that any explicit set of assumptions, no matter how implausible, is preferable to any unknown set of assumptions, no matter how allegedly general. In this case the assumptions are not patently implausible. One can quibble about possible inconsistencies and question whether combat decision making really proceeds from the top down, but, as a whole, the structure seems to be a useful representation of the hierarchical decision structure the model developers wished to incorporate. Whether it is really important to model such a structure, especially at the expense of other aspects such as attrition computation is, of course, quite another matter. Our opinion is that it is not, but the question is at least partly one of taste. There appears to be, in any case, no clear understanding of how the great amount of detail in one part of the model affects the behavior of other parts, or of whether the lack of detail in the other parts entirely or partially negates the detail in the decision-making portion. We suspect, mostly on intuitive and philosophical grounds, that the latter may be true. This possible negation of limited disaggregation is, of course, not peculiar to the CEM.



## 8. ALLOCATION PROCESSES

Most resource allocation processes occurring in the CEM have been described in previous sections. This section describes the allocation of TF aircraft (tactical fighter aircraft) among the missions

- CA = counter-air
- CAS = close air support
- R/I = reconnaissance and interdiction.

The allocation is, somewhat strangely, based mainly on relative losses in the three missions, with the state of the ground campaign affecting the allocation only in extreme cases, as we describe in more detail below. Of the TF aircraft assigned to CA and R/I missions, a user-input fraction function as SF (sweep fighters) in defense of the others.

The aircraft allocation is based on current

- per sortie losses in the CA mission
- per sortie losses in the R/I mission
- theater period losses on air bases

with allocation changes effected by means of user-supplied attrition thresholds and fractional allocation changes. More specifically, the user provides as inputs the three thresholds

- $\alpha_1$  = per sortie attrition threshold for CA mission
- $\alpha_2$  = per sortie attrition threshold for R/I mission
- $\alpha_3$  = per theater period loss threshold for aircraft on air bases.

In terms of current per sortie loss rates  $A_1$ ,  $A_2$ , in the CA and R/I missions, respectively, and current per theater period loss rate  $A_3$  for aircraft on air bases, one of eight cases then obtains:

CA	CAS	R/I
$A_1 < \alpha_1$	$A_2 < \alpha_2$	$A_3 < \alpha_3$
$A_1 < \alpha_1$	$A_2 < \alpha_2$	$A_3 \geq \alpha_3$
$A_1 < \alpha_1$	$A_2 \geq \alpha_2$	$A_3 < \alpha_3$
$A_1 < \alpha_1$	$A_2 \geq \alpha_2$	$A_3 \geq \alpha_3$
$A_1 \geq \alpha_1$	$A_2 < \alpha_2$	$A_3 < \alpha_3$
$A_1 \geq \alpha_1$	$A_2 < \alpha_2$	$A_3 \geq \alpha_3$
$A_1 \geq \alpha_1$	$A_2 \geq \alpha_2$	$A_3 < \alpha_3$
$A_1 \geq \alpha_1$	$A_2 \geq \alpha_2$	$A_3 \geq \alpha_3$

For each of these changes the user must also provide as inputs a set of percentage increments in the aircraft allocation, which sum to zero. If, for example, the current allocation is

$$\begin{aligned} \text{CA} &= 40\% \\ \text{CAS} &= 40\% \\ \text{R/I} &= 20\%, \end{aligned}$$

if the case  $A_1 < \alpha_1$ ,  $A_2 < \alpha_2$ ,  $A_3 < \alpha_3$  obtains and the corresponding input increments are CA : -4%, CAS : +8%, R/I : -4% (which is plausible: if aircraft losses are acceptably low, further aircraft contribution to the ground campaign should be sought), then the new allocation is

$$\begin{aligned} \text{CA} &= 36\% \\ \text{CAS} &= 48\% \\ \text{R/I} &= 16\% . \end{aligned}$$

Some care must be exercised in specification of the allocation increments in order to avoid paradoxical effects. Consider, for example, the case  $A_1 \geq \alpha_1$ ,  $A_2 < \alpha_2$ ,  $A_3 < \alpha_3$  in

which the loss rate threshold for the CA mission is exceeded. Presumably one should provide as corresponding allocation increments values that decrease the allocation of aircraft to the CA mission. But, in view of Equation (24), this may *increase* the per sortie loss rate in that mission. The model would respond by decreasing the percentage of aircraft allocated to the CA mission still further, and so on. Hence this allocation process may be ineffective as a means of keeping aircraft losses acceptably low. This potential anomaly is yet another manifestation of the generally low quality of the air combat portion of the CEM.

The user may also specify maximum and minimum percentages for each role and two backward FEBA movement rates. If the first rate is exceeded, all aircraft assigned to R/I are diverted to CAS, while if the second rate (which is larger) is exceeded, all TF aircraft are assigned to the CAS mission. Otherwise, the evolution of the ground campaign has no direct effect on aircraft allocations.



## 9. INPUTS AND OUTPUTS

In terms of preparation of input data the CEM appears to require somewhat, if not considerably, more effort than comparable models. The method of resource accounting (by maneuver unit status rather than weapon category) entails specification of the structure of every such unit as part of the input preparation process. In other models this specification is either unnecessary or at least partially internally accomplished; in the CEM it may entail substantial labor. Moreover, specification of battalion types and derivation of single weapon firepower potentials are also difficult and time consuming.

The CEM does not appear to differ significantly from comparable models in terms of number and variety of outputs.

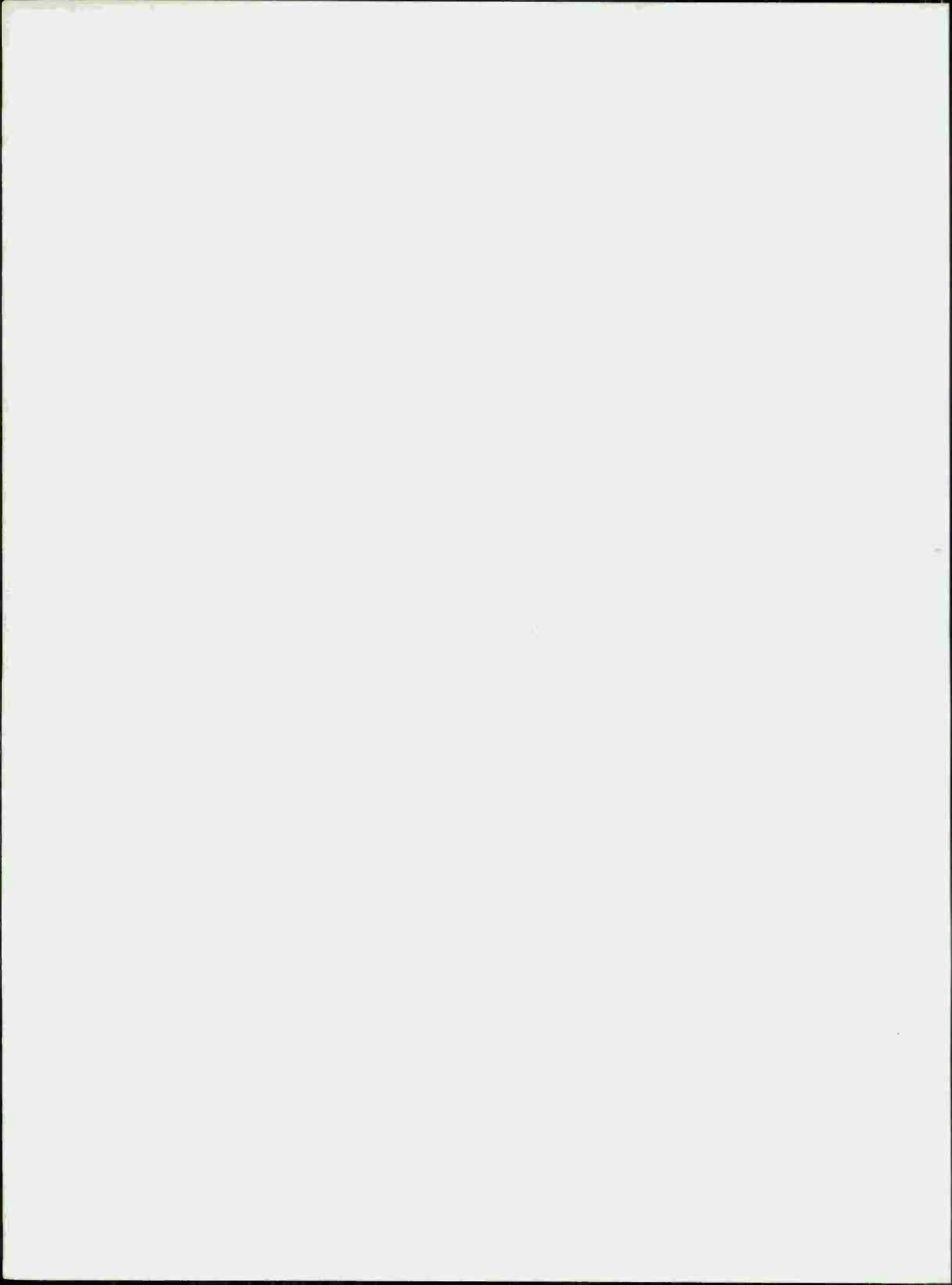


## 10. CONCLUSION

In conclusion, the most positive aspects of the CEM are the detailed representation of hierarchical combat decision making and the method of resource accounting. Both features are not present in comparable models (at least not to the same extent) so the CEM does provide an alternative to such models for certain aspects of combat simulation.

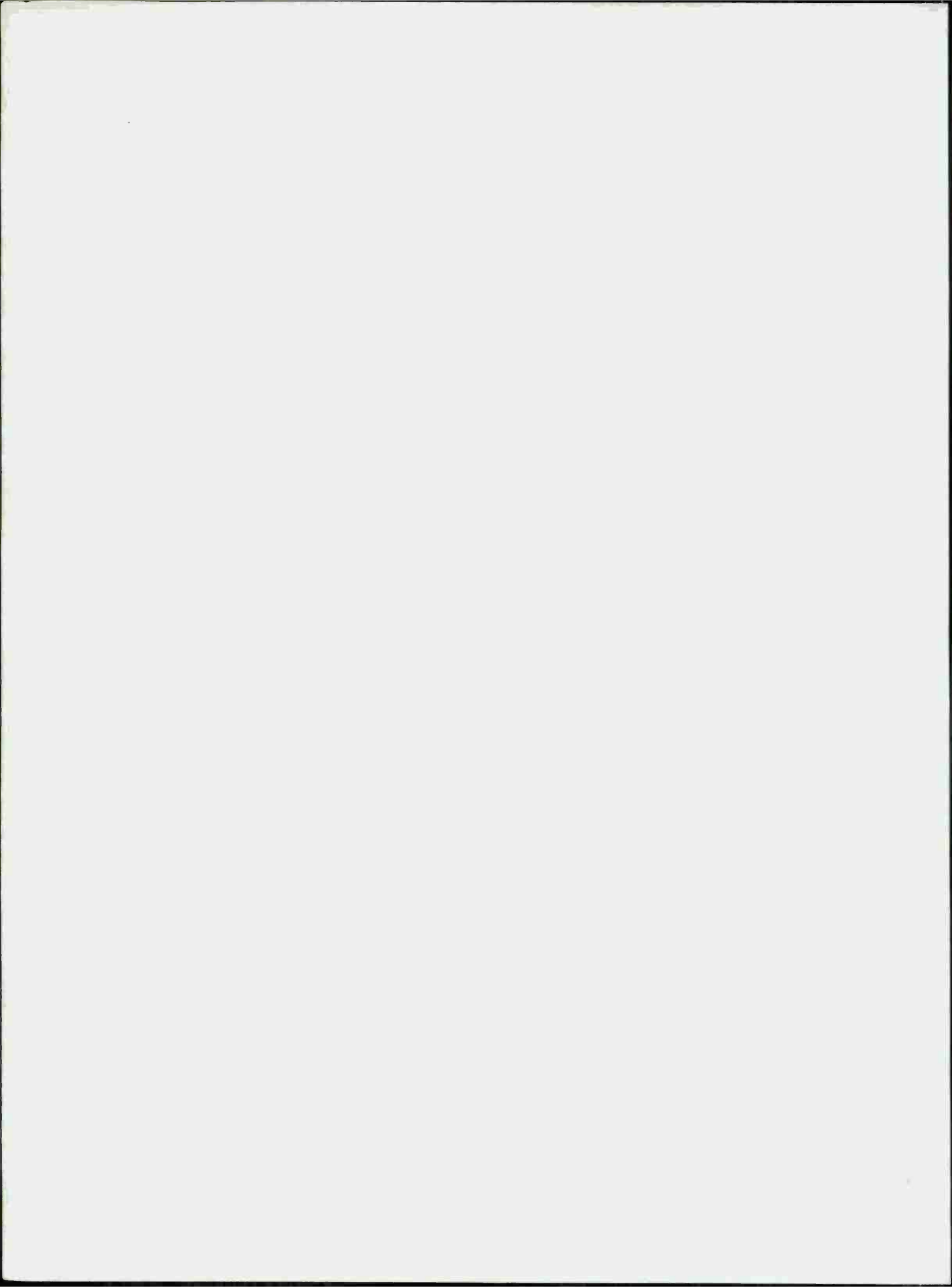
On the negative side, the principal shortcomings of the model are the exclusive use of firepower potentials for computation of both attrition and FEBA movement and a generally poor representation of the air combat portion of the campaign. The latter shortcoming is manifested in attrition equations, the logic of interactions, the level of detail (especially the number of types of aircraft permitted), and the allocation processes.

Finally, the CEM leaves us (more so than comparable models such as IDAGAM I, Lulejian-I, and Vector-I) with a distinct impression of having been developed for the sole purpose of studying potential NATO-Warsaw Pact conventional land campaigns in Europe. This impression arises from the various (real and apparent) asymmetries within the model, from the emphasis on decision-making processes, and from the relative lack of attention to attrition computations and to air combat in general. Such singleness of purpose is both good and bad: good in that the CEM explicitly incorporates certain asymmetries present in the potential situation to which the model is addressed, but bad in the sense that the applicability of the CEM to other analyses may be severely limited.



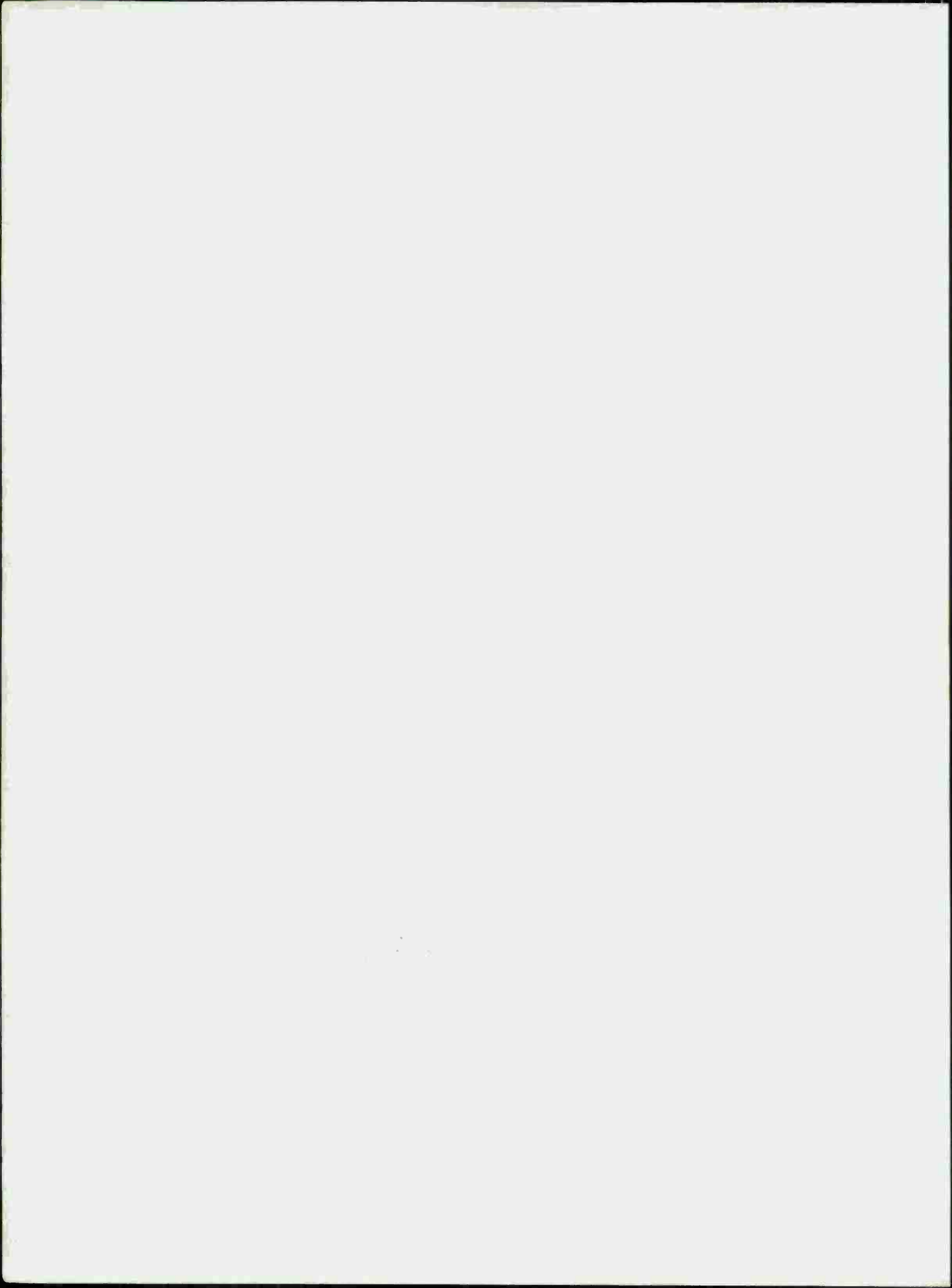
## ACKNOWLEDGMENTS

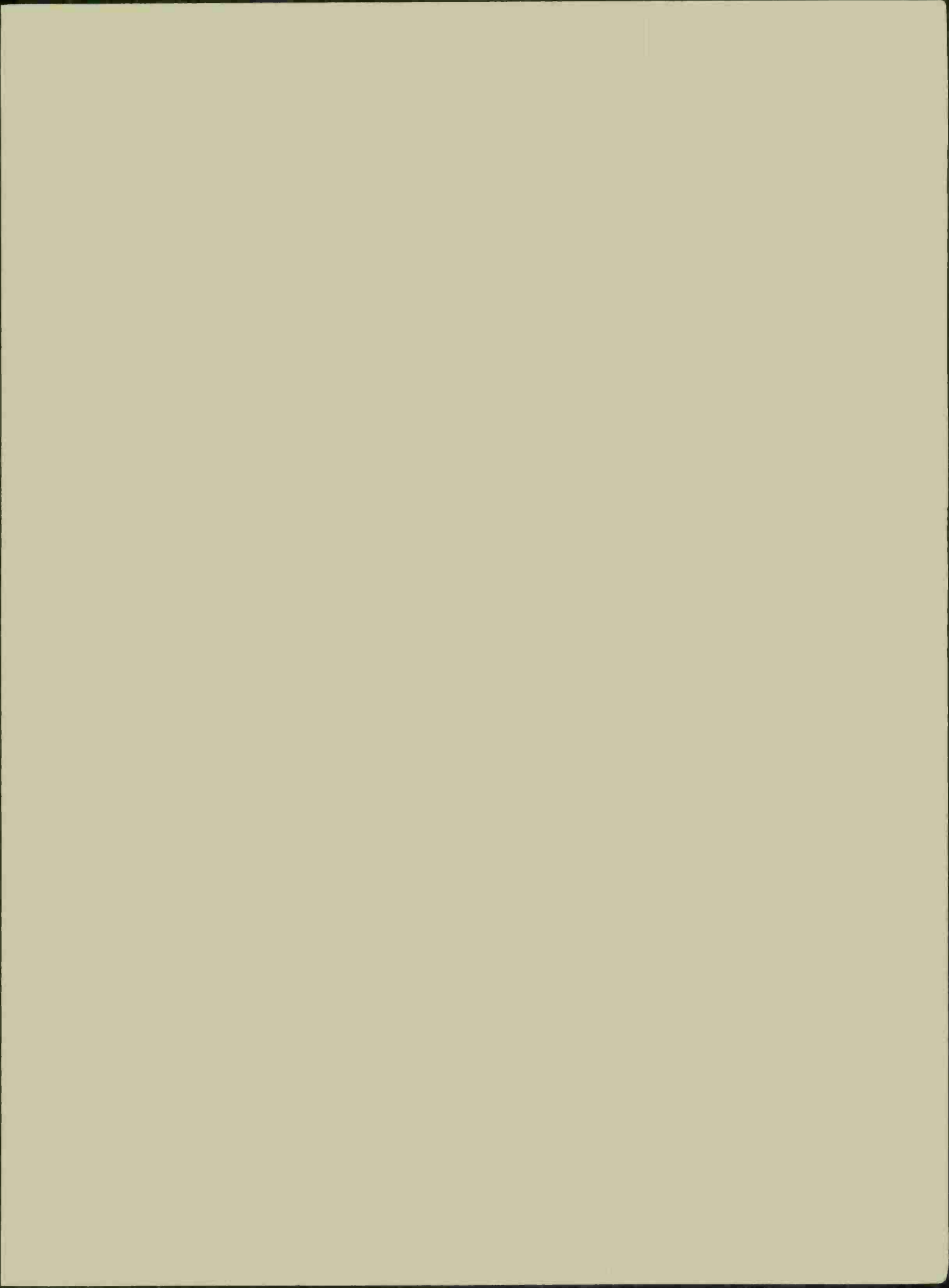
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